New effects in quantum vacuum: photon undulator and transition radiation

2009 J. Phys. A: Math. Theor. 42 375403

(http://iopscience.iop.org/1751-8121/42/37/375403)

IOP PUBLISHING JOURNAL OF PHYSICS A: MATHEMATICAL AND THEORETICAL

J. Phys. A: Math. Theor. **42** (2009) 375403 (9pp) doi:10.1088/1751-8113/42/37/375403

New effects in quantum vacuum: photon undulator and transition radiation

J T Mendonça

CFIF and CFP, Instituto Superior Técnico,1049-001 Lisboa, Portugal

E-mail: titomend@ist.utl.pt

Received 28 May 2009, in final form 16 July 2009
Published 25 August 2009
Online at stacks.iop.org/JPhysA/42/375403

Abstract
We consider a new optical property of quantum vacuum, as predicted by quantum electrodynamics. It is associated with the propagation of an intense laser pulse, in the presence of a periodic static magnetic field. The existence of an effective charge distribution for the intense laser beam is demonstrated. The photon undulator effect results from the spacetime modulations of this effective charge. This is similar to an electron undulator, where the electron beam is replaced by a photon beam. We also discuss a closely related effect, which can be called photon transition radiation in vacuum. It is associated with the effective charge variation at a magnetic boundary. This work could lead to new experimental configurations for quantum vacuum research with future multi-Peta–Watt laser systems.

PACS numbers: 12.20.Ds, 14.70.Bh, 42.50.Xa

1. Introduction

It is well known that vacuum can be considered a nonlinear optical medium, due to the excitation of virtual electron–positron pairs, as predicted by quantum electrodynamics (QED) [1, 2]. However, these nonlinear corrections to the photon dispersion relation in vacuum are extremely weak, and have not been directly observed. Several different experimental tests of such nonlinearities have been proposed. They include photon splitting and birefringence [3, 4], second harmonic generation [5], self-focusing [6] and nonlinear wave mixing in the microwave [7] or in the optical domain [8]. Attention has also recently been given to collective photon phenomena [9], such as the electromagnetic wave collapse [10, 11], photon acceleration in vacuum [12], as well as the formation of photon bullets [13] and light wedges [14].

In the present work, we consider a new class of optical quantum vacuum effects, which are associated with the propagation of an intense laser pulse (or a more arbitrary radiation signal), in the presence of a non-uniform static magnetic field, produced by some external magnetic source. These effects are essentially due to the existence of an effective charge distribution for the intense laser pulse. Such a charge is formally similar to that already demonstrated for photons in a plasma [15] and, in that sense, quantum vacuum could be seen as a kind of virtual plasma, made up of virtual electron–positron pairs. Other effects in magnetized vacuum have also been discussed in the literature, such as photon bending and vacuum lensing [16, 17]. Reference should also be made to experimental work on vacuum magnetic birefringence, as recently discussed in [18].

Here we first demonstrate the existence of photon effective charge in a magnetized vacuum. We will consider an intense photon beam, such as that of an intense laser pulse, propagating in a wiggler magnetic field, similar to those used in free electron laser research. But here the relativistic electron beam will be replaced by the intense laser pulse. The resulting photon undulator effect is a direct consequence of the spacetime modulations of the laser effective charge distribution in vacuum. We will also show that a closely related effect can take place at the boundary between two regions of different vacuum magnetization. This new effect leads to the emission of a burst of broadband radiation, when the laser pulse crosses the magnetic boundary, and can be called photon transition radiation in vacuum.

2. Basic equations

The nonlinear QED effects associated with the creation of virtual electron–positron pairs in vacuum can be described by the Heisenberg–Euler Lagrangian density, which can be written as the usual classical electromagnetic Lagrangian density $\mathcal{L}_0$ plus a nonlinear quantum correction $\delta\mathcal{L}$, of the form [1, 19]

$$\mathcal{L} = \mathcal{L}_0 + \delta\mathcal{L} = \epsilon_0\mathcal{F} + \zeta(4\mathcal{F}^2 + 7\mathcal{G}^2), \tag{1}$$

where the quantities $\mathcal{F}$ and $\mathcal{G}$ are the relativistic invariants defined by $\mathcal{F} = \frac{1}{2}(E^2 - c^2B^2)$ and $\mathcal{G} = c(\vec{E}\cdot\vec{B})$. The nonlinear parameter appearing in (1) is $\zeta = 2\alpha^2\epsilon_0^2\hbar^3\big/45m_e^4c^5$, and $\alpha = e^2/2\epsilon_0 hc \simeq 1/137$ is the fine structure constant. This is valid for sub-critical field intensities, such that the creation of real pairs can be neglected [2]. The resulting Maxwell's equations in vacuum take their usual form, but where the polarization and magnetization vectors resulting from the nonlinear QED corrections included in (1) are given by

$$\vec{P} = 2\zeta(4\mathcal{F}\vec{E} + 7c\mathcal{G}\vec{B}), \qquad \vec{M} = -2c^2\zeta(4\mathcal{F}\vec{B} + 7\mathcal{G}\vec{E}/c). \tag{2}$$

We can then derive a propagation equation for the electric field $\vec{E}$ in the form

$$\left(\nabla^2 - c^{-2}\partial_t^2\right)\vec{E} = \mu_0\left[\partial_t^2\vec{P} + c^2\nabla(\nabla\cdot\vec{P}) + \partial_t(\nabla\times\vec{M})\right], \tag{3}$$

and a similar equation for the magnetic field.

We consider wave propagation in vacuum in the presence of a static magnetic field $\vec{B}_e$ produced by some external source. In order to treat this case, we have to replace in equations (2) and (3) the total magnetic field $\vec{B}$ by $(\vec{B} + \vec{B}_e)$, where the first term represents the wave field. If an intense laser pulse propagates in such a magnetized vacuum, secondary radiation can eventually be emitted. The total electric field is then made of two distinct parts $\vec{E}_0$ and $\vec{E}$, such that $|E_0| \gg |E|$ represents the intense laser field. The same will occur for the wave magnetic field terms $\vec{B}_0$ and $\vec{B}$, with $|B_0| \gg |B|$. We can describe the laser field by

$$\vec{E}_0(\vec{r}, t) = \vec{E}_0(\vec{r}_\perp, \eta)\exp(\mathrm{i}\vec{k}_0\cdot\vec{r} - \mathrm{i}\omega_0 t) + \text{c.c.} \tag{4}$$

where $\vec{E}_0(\vec{r}_\perp, \eta)$ describes the slowly varying pulse envelope, with the axial co-moving variable $\eta = (z - v_0 t)$, and the transverse vector position $\vec{r}_\perp \equiv (x, y)$, for propagation along the z-direction. We also have $\vec{k}_0$ and $\omega_0 = k_0 c$ as the central values of the pulse wavevector and frequency. Here, the pulse group velocity is v_0, nearly equal to but slightly smaller than c, due to the finite transverse dimensions of the laser pulse. It is also obvious that $\vec{B}_0 = E_0\vec{b}_0/c$, with $\vec{b}_0 = \vec{k}_0 \times \vec{e}_0/k_0$, where $\vec{e}_0$ is the unit polarization vector. This implies that $(\vec{E}_0 \cdot \vec{B}_0) = 0$, and $\left(E_0^2 - c^2 B_0^2\right) = 0$, or equivalently $\mathcal{F}_0 = \mathcal{G}_0 = 0$, for the respective relativistic invariants. Under these assumptions, and neglecting the quadratic terms with respect to the secondary radiation field $\vec{E}$, equation (2) can be written in the form

$$\vec{P} = \vec{P}_0 + \vec{P}_r, \qquad \vec{M} = \vec{M}_0 + \vec{M}_r, \tag{5}$$

where the quantities $\vec{P}_0$ and $\vec{M}_0$ are quadratic with respect to the intense laser field $\vec{E}_0$, and the quantities $\vec{P}_r$ and $\vec{M}_r$ are quadratic with respect to the static external magnetic field $\vec{B}_e$ and linear with respect to the radiation field $\vec{E}$. We have

$$\vec{P}_0 = -2\zeta c[2(\vec{b}_0 \cdot \vec{B}_e)\vec{e}_0 - 7(\vec{e}_0 \cdot \vec{B}_e)\vec{b}_0]|E_0(\vec{r}_\perp, \eta)|^2 \tag{6}$$

and

$$\vec{M}_0 = 2\zeta c^2[2(\vec{b}_0 \cdot \vec{B}_e)\vec{b}_0 - 7(\vec{e}_0 \cdot \vec{B}_e)\vec{e}_0]|E_0(\vec{r}_\perp, \eta)|^2. \tag{7}$$

We also have, for the small polarization terms,

$$\vec{P}_r = -2\zeta c^2\big[2B_e^2\vec{E} - 7(\vec{E} \cdot \vec{B}_e)\vec{B}_e\big], \qquad \vec{M}_r = 4\zeta c^2(\vec{B} \cdot \vec{B}_e)\vec{B}_e. \tag{8}$$

Retaining only the dominant terms, we can then write the propagation equation for the secondary radiation field $\vec{E}$ in the form

$$\left(\nabla^2 - c^{-2}\partial_t^2\right)\vec{E} = \mu_0\partial_t\vec{J}_0 - \mu_0 c^2\nabla(\nabla \cdot \vec{P}_0), \tag{9}$$

with the current

$$\vec{J}_0 = \partial_t\vec{P}_0 + \nabla \times \vec{M}_0. \tag{10}$$

These will be the basic equations for our vacuum radiation problem, which have to be applied to specific field geometries.

3. Photon equivalent charge

In the calculation of the source current $\vec{J}_0$, we now assume a geometry where the static external magnetic field is modulated along the direction of propagation of the intense laser field $\vec{E}_0$. Let us call this direction the axis Oz. We then have $\vec{B}_e(\vec{r}) \equiv \vec{B}_e(z)$. We note that both $\vec{M}_0$ and $\vec{P}_0$ are transverse with respect to the intense laser beam propagation. We can then write

$$\nabla \times \vec{M}_0 = \partial_z(\vec{M}_{0\perp} \times \vec{e}_\perp), \tag{11}$$

were $\vec{e}_\perp = (\vec{e}_x, \vec{e}_y)$. Or, in a more explicit form,

$$\nabla \times \vec{M}_0 = -2\zeta c^2\partial_z[2(\vec{b}_0 \cdot \vec{B}_e)\vec{e}_0 - 7(\vec{e}_0 \cdot \vec{B}_e)\vec{b}_0]|E_0(\vec{r}_\perp, \eta)|^2. \tag{12}$$

We should now realize that the laser pulse energy envelope depends on the co-moving coordinate $\eta = z - v_0 t \simeq z - ct$. This means that $(\partial_t + c\partial_z) = (c - v_0)\partial_\eta \simeq 0$. We can then write, for the nonlinear current (10), the following expression:

$$\vec{J}_0 = -2\zeta c^2|E_0(\vec{r}_\perp, \eta)|^2\vec{j}\,(\partial_z B_e), \tag{13}$$

where $\vec{b}_e = \vec{B}_e/B_e$, and

$$\vec{j} = [2(\vec{b}_0 \cdot \vec{b}_e)\vec{e}_0 - 7(\vec{e}_0 \cdot \vec{b}_e)\vec{b}_0]. \tag{14}$$

Let us now estimate the $(\nabla \cdot \vec{P}_0)$ contribution to the secondary field. Noting that $P_{0z} = 0$, and assuming without loss of generality that, for a linearly polarized laser pulse, $\vec{e}_0 = \vec{e}_x$ and $\vec{b}_0 = \vec{e}_y$, we have

$$\nabla \cdot \vec{P}_0 = -2\zeta c[2\partial_x(\vec{b}_0 \cdot \vec{B}_e) - 7\partial_y(\vec{e}_0 \cdot \vec{B}_e)]|E_0(\vec{r}_\perp, \eta)|^2. \tag{15}$$

We can easily understand that, for a laser pulse with a finite transverse width, or an external magnetic field with a transverse intensity profile, this would lead to an additional contribution to the radiation field. However, such a contribution will be considered negligible here, for simplicity. Such an approximation is justified if we neglect the static magnetic field transverse inhomogeneity over the laser waist, $\partial_\perp B_e \ll \partial_z B_e$. On the other hand, it can be shown that the remaining terms will give a contribution of order $\lambda/a \ll 1$ to the propagation equation (9), where λ is the wavelength of the emitted secondary radiation (to be discussed below) and a is the laser beam waist. The equation of propagation for the secondary radiation field will then reduce to

$$\left(\nabla^2 - c^{-2}\partial_t^2\right)\vec{E} = \mu_0 c \partial_t Q(\vec{r}_\perp, \eta)\vec{j}, \tag{16}$$

with

$$Q(\vec{r}_\perp, \eta) = -2\zeta c|E_0(\vec{r}_\perp, \eta)|^2 (\partial_z B_e) . \tag{17}$$

This quantity $Q(\vec{e}_\perp, \eta)$ has the dimensions of an electric charge distribution. This can be seen as the equivalent charge associated with the intense laser pulse propagating in vacuum. We can give it a conceptually more interesting form, by introducing the photon occupation number (or photon number density), as defined by [20]

$$N_0(\vec{r}_\perp, \eta) = \frac{\epsilon_0}{4\hbar\omega_0}|E_0(\vec{r}_\perp, \eta)|^2, \tag{18}$$

where the laser field spectrum is supposed to be nearly monochromatic with mean frequency ω_0. Note that, in this expression, we could have included the vacuum contribution $N_0 = 1/2$, but this is irrelevant here because we are dealing with very intense fields, such that $N_0 \gg 1$. The nonlinear current $\vec{J}_0$ can then be rewritten as

$$\vec{J}_0 = cq_0 N_0(\vec{r}_\perp, \eta)\frac{\vec{j}}{|j|}, \tag{19}$$

where

$$q_0 = -\frac{\epsilon_0 \zeta c}{2\hbar\omega_0}|j|\partial_z B_e \tag{20}$$

is the effective single photon charge due to the vacuum nonlinearities. This is qualitatively very similar to the equivalent photon charge in a plasma already discussed in the literature [15, 20]. Here the disturbed vacuum state can be seen as a virtual plasma, made of virtual electron–positron pairs. But, in contrast with the real plasma case, this effective photon charge can only exist in the presence of the inhomogeneous static magnetic field. Such a difference is due to the different nature of the vacuum and real plasma nonlinearities.

4. Photon undulator

Let us now try to solve the above nonlinear wave equation for the particular case of a wiggler magnetic field, with periodicity λ_w in the z-direction. It can be described by

$$\vec{B}_e(\vec{r}) = \vec{B}_{0e}\sin(k_w z), \tag{21}$$

where $k_w = 2\pi/\lambda_w$. It is clear that, if the laser field length is of the order or smaller than the wiggler period (which is valid for pulse durations $\tau \leqslant \lambda_w/c$), its effective charge will oscillate along the pulse propagation, therefore leading to the emission of secondary radiation. In order to determine its intensity and spectrum, we consider a double Fourier transformation of the radiation field, both in time and in the transverse space variable, as defined by

$$\vec{E}(\vec{r}, t) = \int\frac{d\omega}{2\pi}\int\frac{d\vec{k}_\perp}{(2\pi)^2}\vec{E}_\omega(\vec{k}_\perp, z)\exp(-i\omega t + i\vec{k}_\perp\cdot\vec{r}_\perp). \tag{22}$$

Replacing this in equation (16), we get

$$\left(\partial_z^2 - k_\perp^2 + \frac{\omega^2}{c^2}\right) E_\omega(\vec{k}_\perp, z) = -\mathrm{i}\omega\mu_0 c(\vec{j}\cdot\vec{e}_\omega) \int_{-\infty}^{\infty} \mathrm{d}t \int \mathrm{d}\vec{r}_\perp Q(\vec{r}_\perp, \eta) \exp\left(\mathrm{i}\omega t - \mathrm{i}\vec{k}_\perp\cdot\vec{r}_\perp\right). \tag{23}$$

where we have introduced the unit polarization vector $\vec{e}_\omega$ for the radiation field component $\vec{E}_\omega(\vec{k}_\perp, z)$. For a generic transverse or radial profile of the intense laser pulse, we can write

$$|E_0(\vec{r}_\perp, \eta)|^2 = f(\vec{r}_\perp)|E_0(\eta)|^2, \qquad \int f(\vec{r}_\perp)\,\mathrm{d}\vec{r}_\perp = 1. \tag{24}$$

where $f(\vec{r}_\perp)$ is a given normalized profile function, for instance a Gaussian. Using equations (21) and (24), we can then define an auxiliary function, such that

$$g_\omega(\eta, \vec{k}_\perp, t) = a_\omega(\vec{k}_\perp) k_w B_e |E_0(\eta)|^2 \cos(k_w z), \tag{25}$$

where

$$a_\omega(\vec{k}_\perp) = 2\zeta\omega\mu_0 c^2(\vec{j}\cdot\vec{e}_\omega) \int f(\vec{r}_\perp) \exp\left(-\mathrm{i}\vec{k}_\perp\cdot\vec{r}_\perp\right) \mathrm{d}\vec{r}_\perp. \tag{26}$$

For simplicity, we have neglected the variation of the static magnetic field B_e in the transverse direction. Equation (23) can then be written in a simpler and more appropriate form as

$$\left(\partial_z^2 + k^2\right) E_\omega(z) = \mathrm{i}\int_{-\infty}^{\infty} g_\omega(\eta, \vec{k}_\perp, t) \exp(\mathrm{i}\omega t)\,\mathrm{d}t, \tag{27}$$

with $k^2 = (\omega/c)^2 - k_\perp^2$, and $E_\omega(z) \equiv E_\omega(\vec{k}_\perp, z)$. Integrating this equation, we can write for fields propagating in the forward (positive z-direction) the following solution:

$$E_\omega(z) = -\frac{1}{2k}\,\mathrm{e}^{\mathrm{i}kz} \int_{-\infty}^{\infty} \mathrm{d}t\,\mathrm{e}^{\mathrm{i}\omega t} \int_{-\infty}^{z} \mathrm{d}z' g_\omega(\eta', \vec{k}_\perp, t)\,\mathrm{e}^{-\mathrm{i}kz'}, \tag{28}$$

where $\eta' = z' - ct$. This solution can also be written as

$$E_\omega(z) = -a_\omega(\vec{k}_\perp) B_e \frac{k_w}{2k}\,\mathrm{e}^{\mathrm{i}kz} \int_{-\infty}^{\infty} \mathrm{d}t\,\mathrm{e}^{\mathrm{i}\omega t} \int_{-\infty}^{z} \mathrm{d}z' |E_0(\eta')|^2 \cos(k_w z')\,\mathrm{e}^{-\mathrm{i}kz'}. \tag{29}$$

This leads to the following asymptotic value, observed at infinity in the forward direction

$$E_\omega(z\to\infty) = -a_\omega(\vec{k}_\perp) B_e \frac{\pi k_w}{2k}\,\mathrm{e}^{\mathrm{i}kz} \int_{-\infty}^{\infty} |E_0(\eta)|^2 [\mathrm{e}^{-\mathrm{i}(k-k_w)\eta}\delta(\omega-\omega_-) + \mathrm{e}^{-\mathrm{i}(k+k_w)\eta}\delta(\omega-\omega_+)]\mathrm{d}\eta, \tag{30}$$

where we used the frequencies

$$\omega_\pm = (k \pm k_w)\beta c. \tag{31}$$

where $\beta = v/c \sim 1$ is the relative group velocity of the intense laser pulse. Only for a plane wave with an infinite transverse width we will exactly get $\beta = 1$. It should be noted that the delta functions in equation (30) will disappear for a wiggler field with finite length L_w. In this case, the radiation spectrum will have a finite bandwidth, of the order of $\Delta\omega_\pm \simeq c/L_w$, which will be negligible for $L_w \gg \lambda_w$.

Let us now consider a laser pulse shape with an axial profile similar to that given by equation (24), but now describing the pulse shape in the z-direction, as defined by $|E_0(\eta)|^2 = |E_0|^2 f_z(\eta)$. The asymptotic radiation field, observed at $z \to \infty$, will then be given by

$$E_\omega(z) = A_\omega\,\mathrm{e}^{\mathrm{i}kz}[f_z(k-k_w)\delta(\omega-\omega_-) + f_z(k+k_w)\delta(\omega-\omega_+)], \tag{32}$$

where the field amplitude is determined by

$$A_\omega = -a_\omega(\vec{k}_\perp) B_e \frac{\pi k_w}{2k} |E_0|^2, \tag{33}$$

and $f_z(k \pm k_w)$ are the Fourier components of the axial profile function, such as

$$f_z(k \pm k_w) = \int_{-\infty}^{\infty} f_z(\eta)\, \mathrm{e}^{-\mathrm{i}(k \pm k_w)\eta}\, \mathrm{d}\eta. \tag{34}$$

In the particularly interesting case of a Gaussian pulse with duration τ, defined by $f_z(\eta) = \exp(-\eta^2/2\tau^2c^2)$, we simply have

$$f_z(k \pm k_w) = \sqrt{2\pi}\,\tau c \exp\left[-(k \pm k_w)^2 \frac{\tau^2 c^2}{2}\right]. \tag{35}$$

For a pulse length much shorter than the wiggler period, we can use the approximate values of $f_z(\eta) \simeq \delta(\eta)$ and $f_z(k \pm k_w) \simeq 1$, which is useful for simple estimates. As we can see, the amplitude of the secondary radiation field, resulting from the intense pulse propagation along the magnetic wiggler structure, can be estimated by

$$|E_\omega| \simeq a_\omega |B_e| \frac{\pi k_w}{2k} |E_0|^2. \tag{36}$$

We should note that the frequencies of the secondary radiation are only implicitly determined by equation (31). In order to obtain an explicit value, we have to replace k by its definition, in terms of ω. The result is

$$\omega = \frac{k_w \beta c}{(1 - \beta \cos\theta)}, \tag{37}$$

where θ is the angle of radiation emission, with respect to the axis of the magnetic wiggler field, and is such that $\tan\theta = k_\perp/k$. For nearly axial propagation $\theta \simeq 0$, this can also be written in the form

$$\omega = 2\gamma_{\text{eff}}^2 k_w c, \qquad \gamma_{\text{eff}} \simeq \frac{1}{\sqrt{1 - \beta^2 \cos^2\theta}}. \tag{38}$$

This shows a remarkable similarity with the frequency emitted by free electron lasers, where the effective gamma factor of the intense laser pulse considered here is replaced by the gamma factor of the relativistic electron beam. Note that we can achieve emission of very high-energy photons, such that $\omega \gg k_w c$. However, the present Heisenberg–Euler model brakes down for frequencies approaching the Compton frequency, $\omega_C = m_e c^2/\hbar$. We can conclude that very high-frequency photons can be emitted along the z-axis, and suggests that enhancement of electron–positron pair creation can eventually take place in this configuration, for high enough values of γ_{eff}.

We should also consider the possible emission of backward radiation, with propagation along the negative z-direction. A similar asymptotic solution, symmetric to that discussed above, but with k replaced by $-k$ in equations (30) and (31), will then be obtained for $z \to -\infty$. In this case, the new emitted frequencies will be determined by

$$\omega'_\pm = \pm \frac{k_w c}{(1 + |\cos\theta|)}. \tag{39}$$

A purely axial signal, with $k_\perp \simeq 0$ and $\omega' \simeq k_w c/2$, will then be expected to propagate in the backward direction. This can be another distinctive feature for the observation of secondary radiation. We should however note that, for background radiation, the contributions of $\nabla \cdot \vec{P}_0$ to the wave equation (9) would eventually become relevant.

5. Photon transition radiation

Let us now consider a closely related type of radiation, produced by an intense laser pulse when it crosses the boundary between a magnetized and a non-magnetized vacuum region. This can be called photon transition radiation. Such a magnetic boundary could be described by

$$\vec{B}_e(z) = \frac{B_{e0}}{2}[1 - \tanh(k_b z)], \tag{40}$$

where $1/k_b$ gives the scale length of the boundary region. When an intense laser pulse propagating along the z-direction crosses such a boundary, an effective electric charge suddenly appears and vanishes. This moving charge will be associated with the effective charge distribution

$$Q(\vec{r}_\perp, \eta) = \zeta c |E_0(\vec{r}_\perp, \eta)|^2 B_{e0} k_b \, \mathrm{sech}^2(k_b z). \tag{41}$$

The radiation field in the forward direction will then be determined by

$$E_\omega(z) = -\frac{k_b}{4k} a_\omega(\vec{k}_\perp) B_{e0} \, \mathrm{e}^{\mathrm{i}kz} \int_{-\infty}^{\infty} \mathrm{d}t \, \mathrm{e}^{\mathrm{i}\omega t} \int_{-\infty}^{z} \mathrm{d}z' |E_0(\eta')|^2 \, \mathrm{sech}^2(k_b z') \, \mathrm{e}^{-\mathrm{i}kz'}. \tag{42}$$

As an illustrative example, we consider the case of a sharp magnetic boundary, such that $k_b \gg \tau c$, where τ is the intense laser pulse duration. We can then replace equation (40) by $\vec{B}_e(z) = B_{e0} H(-z)$, where $H(z)$ is the Heaviside function. In the expression for the effective charge distribution (41), the hyperbolic square function will then be replaced by a delta function (multiplied by a factor of 2), and the corresponding radiation field will be given, far away from the boundary, by

$$E_\omega(z) = -\frac{k_b}{2k} a_\omega(\vec{k}_\perp) B_{e0} \, \mathrm{e}^{\mathrm{i}kz} \int_{-\infty}^{\infty} |E_0(t, z = 0)|^2 \, \mathrm{e}^{\mathrm{i}\omega t} \, \mathrm{d}t. \tag{43}$$

For a Gaussian laser pulse, this will lead to a burst of broadband radiation, emitted when the laser pulse crosses the magnetic boundary, as described by

$$E_\omega(z) = -\sqrt{\frac{\pi}{2}} \frac{k_b}{k} a_\omega(\vec{k}_\perp) B_{e0} |E_0|^2 \tau \exp\left(-\frac{1}{2}\omega^2\tau^2\right) \mathrm{e}^{\mathrm{i}kz}. \tag{44}$$

We can see that the shorter the laser pulse the larger will be the secondary radiation spectrum but also the weaker will be the intensity of its spectral components, as expected.

6. Conclusions

In this paper, a new class of nonlinear effects associated with intense electromagnetic radiation propagating in vacuum in the presence of a static magnetic field was considered. This is associated with the existence of an effective photon charge, due to the excitation of virtual electron–positron pairs, as predicted by quantum electrodynamics. We have shown that such an effective charge is similar to that already discovered for photons in a real plasma [15]. But, in contrast with a real plasma, the effective photon charge in the nonlinear quantum vacuum can only exist in the regions of magnetic field inhomogeneity. When an intense laser pulse propagates in a magnetized vacuum, its effective charge distribution varies in space and time, due to the changes of the local magnetic field, thus leading to secondary radiation.

Two particular examples of static non-uniform magnetic fields were considered. First, we have assumed a wiggler field, similar to that used in free electron laser research, where the static field amplitude is modulated in the forward direction. We have shown that, in this case, a photon undulator effect can occur, leading to the emission of secondary radiation with a well-defined frequency spectrum. This effect is of the same order of magnitude as that of the other vacuum effects mentioned in the introduction. In particular, it is closely connected with harmonic generation, because it results from the same nonlinear terms (slowly varying here, instead of the rapidly oscillating second-order terms previously considered in the literature [5]).

Apart from its intrinsic value as a new process, it is also useful to compare the present radiation mechanism with that of [8], where four-wave mixing was considered. This is a typical experimental proposal using ultra-intense laser systems, such as the Astra Gemini laser [21]. Two laser beams of 0.5PW can be used at an 800nm wavelength. The proposed experimental configuration also includes the use of a third beam of 0.1PW, leading to an expected excitation of secondary emission of 0.07 number of photons per shot, which is marginally poor experimental observation. In that case, three laser beams with frequencies and field amplitudes ω_j and E_j, with $j = 1, 2, 3$, are mixed at right angles to create the field E_4 at frequency ω_4. The field amplitude predicted by the present undulator process compares with wave mixing process as $|E/E_4| \simeq (\omega/\omega_4)|(cB_w/E_1)(E_0/E_2)|$. In the proposed experiments, we typically have $E_1 \simeq E_0$ and $E_2 \simeq 10^{-1}E_0$. Assuming emitted frequencies in the range $\omega \geqslant 10$–$10^2\omega_4$, the undulator process will then be more efficient than wave mixing if $cB_w \simeq 10^{-2}$–$10^{-3}E_1$. Such high magnetic fields are not achieved with the present wiggler structures, but could be generated by ultra-intense intense lasers. This suggests that the present process can be at least as efficient as that of [8], with the additional experimental advantage of producing photons in a very different frequency range.

Second, we have shown that photon transition radiation can occur when the intense laser pulse crosses the boundary between a magnetized and a non-magnetized region of vacuum. A sudden variation of the effective charge distribution will then take place, leading to the emission of a broadband burst of radiation. This new effect at a vacuum boundary is similar to the photon transition radiation at a plasma boundary [22]. Here again, the quantum vacuum behaves as a virtual plasma, made up of virtual electron–positron pairs. On the other hand, this can also be considered as the analogue of the usual bremsstrahlung, where the acceleration (or deceleration) of the charged particles is replaced by a temporal change on the photon effective charge distribution.

In conclusion, three different properties of quantum vacuum were analysed here for the first time: photon effective charge, photon undulator emission and photon transition radiation. A final note concerns the symmetry of the electric and magnetic fields in the Heisenberg–Euler Lagrangian density. For a non-magnetized vacuum, and given such a symmetry, similar effects associated with a static electric field should be expected. A non-uniform electric field will also induce an effective photon charge. Consequently, an intense laser pulse propagating in a modulated electric field will be able to excite secondary undulator emission. Likewise, an electric field boundary will produce a burst of photon transition radiation. This work could stimulate future quantum vacuum experiments using multi-Peta–Watt laser systems [23]. Here we have neglected the zero-point fluctuations of spinor fields with mass m_e in a magnetic field [24], which could eventually lead to additional contributions. Finally, the existence of the effective photon charge could eventually lead to photon cyclotron motion, which is clearly linked with photon bending and vacuum lensing. This is an interesting problem to be addressed in the future.

References

[1] Heisenberg W and Euler H 1936 *Z. Phys.* **98** 714
[2] Schwinger J 1951 *Phys. Rev.* **82** 664

[3] Bialynicka-Birula Z and Bialynicka-Birula I 1970 *Phys. Rev.* D **2** 2341
[4] Adler S 1971 *Ann. Phys.* **67** 599
[5] Kaplan A E and Ding Y J 2000 *Phys. Rev.* A **62** 043805
[6] Soljacic M and Segev M 2000 *Phys. Rev.* A **62** 043817
[7] Brodin G, Marklund M and Stenflo L 2001 *Phys. Rev. Lett.* **87** 171801
[8] Lundstrom E, Brodin G, Lundin L, Marklund M, Bingham R, Collier J, Mendonça J T and Norreys P 2006 *Phys. Rev. Lett.* **96** 083602
[9] Marklund M and Shukla P K 2006 *Rev. Mod. Phys.* **78** 591
[10] Marklund M, Brodin G and Stenflo L 2003 *Phys. Rev. Lett.* **91** 163601
[11] Shukla P K and Eliasson B 2004 *Phys. Rev. Lett.* **92** 073601
[12] Mendonça J T, Marklund M, Shukla P K and Brodin G 2006 *Phys. Lett.* A **359** 700
[13] Marklund M, Eliasson B and Shukla P K 2004 *JETP Lett.* **79** 208
[14] Shukla P K, Marklund M, Tskhakaya D and Eliasson B 2004 *Phys. Plasmas* **11** 3787
[15] Mendonça J T, Silva L O, Bingham R, Tsintsadze N L, Shukla P K and Dawson J M 1998 *Phys. Lett.* A **239** 373
[16] Jones R V 1961 *Proc. R. Soc.* A **260** 47
[17] Dupays A, Robilliard C, Rizzo C and Bignami G F 2005 *Phys. Rev. Lett.* **94** 161101
[18] Battesti R *et al* 2008 *Eur. Phys. J. D* **46** 323
[19] Itzykson C and Zuber J-B 1980 *Quantum Field Theory* (New York: McGraw-Hill)
[20] Mendonça J T 2001 *Theory of Photon Acceleration* (Bristol: Institute of Physics Publishing)
[21] Collier J *et al* 2004 *Central Laser Facility RAL Annual Report* RAL-TR-2004-025
[22] Mendonça J T, Shukla P K, Bingham R and Tsintsadze N L 1999 *Phys. Scripta* T **82** 125
[23] Mourou G A, Tajima T and Bulanov S V 2006 *Rev. Mod. Phys.* **78** 309
[24] Scandurra M 2000 *Phys. Rev.* D **62** 085024